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Question Number	Scheme	Marks
1. (*a*)	$\text{P}(B) = \text{P}(B \cap D) + \text{P}(B \cap F) + \text{P}(B \cap R)$	
	$= 0.3 \times 0.8 + 0.6 \times 0.6 + 0.1 \times 0.1$	M1
	$= 0.61$	A1 (2)
(*b*)	$\text{P}(R \mid B) = \dfrac{P(R \cap B)}{P(B)} =, \dfrac{0.01}{0.61} = \dfrac{1}{61}$ (accept 0.016 or 0.0164)	M1, A1 (2)
(*c*)	$\dfrac{1}{61} + \dfrac{1}{61} - \dfrac{1}{61}^2$ or $1 - \left(1 - \dfrac{1}{61}\right)^2$	M1 M1
	$= 0.0325$ awrt 0.0325	A1
		(7 marks)
2. (*a*)	$\text{f}(x) = \begin{cases} \frac{3}{7}\text{e}^{-\frac{3}{7}x} & [x \geq 0] \\ [0 & \text{otherwise}] \end{cases}$	M1 A1 (2)
(*b*)	$\text{P}(2 < X < 3) = \displaystyle\int_2^3 \tfrac{3}{7} e^{-\frac{3}{7}x}\,\text{d}x$	M1
	$= \left[-\text{e}^{-\frac{3}{7}x}\right]_2^3$	A1
	$= \text{e}^{-\frac{6}{7}} - \text{e}^{-\frac{9}{7}} = 0.14791$ awrt 0.148	A1 (3)
(*c*)	$\text{P}(X \geq 7) = \left[-\text{e}^{-\frac{3}{7}x}\right]_7^\infty$	M1
	$= \text{e}^{-3} = 0.049787$ awrt 0.050	A1 (2)
		(7 marks)

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Question Number	Scheme	Marks
3. (a)	Geometric	B1 (1)
(b)	$p = \frac{1}{8}$	B1
	$\text{P}(S = 5) = \left(\frac{7}{8}\right)^4 \times \left(\frac{1}{8}\right)$	M1
	$= 0.073$	A1 (3)
(c)	$\text{P}(S \geq 3) = (1 - p)^2$	M1 A1 ft
	$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$ awrt 0.766	A1 (3)
(d)	Assume shots are *independent* and *probability* of hits is *constant*	B1 B1 (2)
		(9 marks)
4. (a)	$\text{M}_X(t) = \text{E}(\text{e}^{tx}) = \int_0^a \text{e}^{tx} \frac{1}{a} \text{d}x$	M1
	$= \left[\frac{1}{at}\text{e}^{tx}\right]_0^a$	M1
	$= \left(\frac{\text{e}^{at}}{at}\right) - \left(\frac{1}{at}\right)$	M1
	$= \frac{\text{e}^{at} - 1}{at}$	A1 cso (4)
(b)	$\text{M}_Y(0) - 1 \Rightarrow 1 = \frac{1}{4}(1 + A + B)$ or $A + B = 3$ (1)	M1
	$\text{M}'_Y(t) = \frac{1}{4}(A\text{e}^t + 2B\text{e}^{2t})$	M1
	$\text{E}(Y) = \text{M}'_Y(0) \Rightarrow \frac{5}{4} = \frac{A}{8} + \frac{2B}{4}$ or $A + 2B = 5$ (2)	A1
	$(2) - (1) \Rightarrow B = 2$ and $A = 1$	M1 A1 A1 (6)
(c)	$\text{M}_Z(t) = \text{M}_X(t) \times \text{M}_Y(t) = \frac{\text{e}^{at} - 1}{at} \times \frac{1 + \text{e}^t + 2\text{e}^{2t}}{4}$	B1 ft (1)
		(11 marks)

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Question Number		Scheme		Marks	
5.	(a)	$60 = \dfrac{3(1-p)}{p^2}$		M1 A1	
		$20p = 1 - p$		M1	
		$(5p - 1)(4p + 1) = 0$		M1	
		$p = \frac{1}{5}$		A1	(5)
	(b)	$\mathrm{P}(Y = 8) = \binom{7}{2} p^2 (1-p)^5 \times p$		M1	
		$= \binom{7}{2} \times \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right)^5 = 0.05505$	awrt 0.055	A1	(2)
	(c)	$\mathrm{P}(Y \le 10 \mid ...) = 1 - \mathrm{P}(Y \ge 11 \mid \text{1st head on 2nd toss})$		M1	
		$= 1 - \mathrm{P}(\text{0 heads in 8 tosses}) - \mathrm{P}(\text{1 head in 8 tosses})$		M1	
		$= 1 - 0.8^8 - 8 \times 0.2 \times (0.8)^7$		A1 A1	
		$= 0.49688$	awrt 0.497	A1	(5)
					(12 marks)
6.	(a)	$\mathrm{P(Accept)} = \mathrm{P}(X \le 1 \mid X \sim \mathrm{B}(20, p))$		M1	
		$= (1-p)^{20} + 20(1-p)^{19}p$		M1	
		$= (1-p)^{19}(1 + 19p)$ (*)		A1 cso	(3)
	(b)	$j = 0.880, \quad k = 0.587$		B1, B1	(2)
	(c)	Graph	axes and scales	B1	
			points	B1	
			OC curve	B1	(3)
	(d)(i)	$p = 0.015 \Rightarrow P = 0.96 \Rightarrow \mathrm{P(Reject)} = 0.04$		B1	
	(ii)	$p = 0.065 \Rightarrow P = 0.62 \Rightarrow \mathrm{P(Reject)} = 0.38$		M1 A1	(3)
	(e)	High *probability of acceptance for low p is OK*			
		but not *very efficient* since *negative gradient is not steep enough*		B1 B1	(2)
					(13 marks)

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Question Number	Scheme	Marks
7. (*a*)(i)	$P(X = 2) = \dfrac{e^{-3} \times 3^2}{2!} = 4.5e^{-3}$	M1 A1
(ii)	$P(X \geq 4) = 1 - P(X \leq 3), \quad = 1 - e^{-3}\left(1 + 3 + \dfrac{3^2}{2!} + \dfrac{3^3}{3!}\right)$	M1, A1
	$= 1 - 13e^{-3}$	A1 (5)
(*b*)	y: 0 1 2 3 4	
	x: 0 1 2 3 ≥ 4	
	$P(Y = y)$: e^{-3} $3e^{-3}$ $4.5e^{-3}$ $4.5e^{-3}$ $1 - 13e^{-3}$	B1
	$G_Y(t) = e^{-3}(t^0 + 3t + 4.5t^2 + 4.5t^3) + (1 - 13e^{-3})t^4$	M1
	$= e^{-3}(1 + 3t + 4.5t^2 + 4.5t^3 - 13t^4 \quad (*)$	A1 cso (3)
(*c*)	$G'_Y(t) = e^{-3}(3 + 9t + 13.5t^2 - 52t^3 + 4t^4$	M1 A1
	$\mu = E(Y) = G'_Y(1) = 4 - 26.5e^{-3}$ or 2.68	A1
	$G''_Y(t) = e^{-3}(9 + 27t - 156t^2) + 12t^2$	M1 A1
	$G''_Y(1) = e^{-3}(-120) + 12 = 12 - 120e^{-3}$	A1
	$\sigma^2 = G''_Y(1) + G'_Y(1) - [G'_Y(1)]^2 \quad (= 1.52...)$	M1
	$\sigma = \sqrt{\sigma^2} = 1.23$	A1(8)
		(15 marks)